



Conceptual Understanding of Students with Special Needs in Two-dimensional Shapes Based on Pirie Kieren's Theory

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ABSTRACT

This study aims to describe the conceptual understanding of two-dimensional shapes in students with special needs based on Pirie Kieren's theory in terms of learning motivation. This research is a case study with the research subject is 1 students with special needs of class IV at SDN Pademawu Timur V The results of this study are low learning motivation with layers of understanding on the square starting from Image making then head to property noticing to ineffective folding back to image making and do ineffective folding back again to primitive knowing. In solving the square problem, the subject doubled back to ineffective image making and primitive knowing. Meanwhile, in the rectangular problem, the subject does one folding back to image making. There are similarities in solving square and rectangular problems, for example the subject describes the two-dimensional shape first before image making and folding back because they don't know how to calculate the area of a square and rectangle. From the similarities and differences in the thinking flow of students with special needs in squares and rectangles, educators are expected to be able to pay attention to their understanding abilities and learning motivation before carrying out learning.

Keywords: *conceptual understanding, student with special needs, Pirie Kieren's theory.*

ABSTRAK

Penelitian ini bertujuan untuk mendeskripsikan pemahaman konsep bangun datar atau dua dimensi siswa berkebutuhan khusus berdasarkan teori Pirie Kieren ditinjau dari motivasi belajar. Penelitian ini merupakan studi kasus dengan subjek penelitian adalah 1 orang siswa berkebutuhan khusus kelas IV SDN Pademawu Timur V. Hasil penelitian ini adalah motivasi belajar yang rendah dengan lapisan pemahaman tentang persegi dimulai dari *image making* kemudian dilanjutkan ke *property noticing* hingga kembali lagi (*folding back*) ke *image making* dan kemudian kembali lagi ke *primitive knowing*. Pemahaman tentang persegi panjang dimulai dari *image making* kemudian dilanjutkan ke *property noticing* yang tidak efektif kemudian kembali lagi (*folding back*) ke *image making*. Dalam menyelesaikan masalah persegi, subjek melakukan dua kali *folding back* ke *image making* yang tidak efektif dan *primitive knowing*. Sedangkan pada masalah persegi panjang, subjek melakukan satu kali *folding back* ke *image making*. Terdapat kesamaan dalam menyelesaikan masalah persegi dan persegi panjang, misalnya subjek mendeskripsikan bangun dua dimensi terlebih dahulu sebelum *image making* dan *folding back* karena tidak mengetahui cara menghitung luas persegi dan persegi panjang. Dari persamaan dan perbedaan alur berpikir siswa berkebutuhan khusus pada persegi dan persegi panjang, pendidik diharapkan mampu memperhatikan kemampuan pemahaman dan motivasi belajar yang dimiliki sebelum melaksanakan pembelajaran.

Kata Kunci: pemahaman konseptual, siswa berkebutuhan khusus, teori Pirie Kieren.

INTRODUCTION

Education is an effort to prepare a person to be able to solve life problems in the present and the future (Djumali et al., 2014). One of the subjects that makes someone learn to solve problems is mathematics. Mathematics is a science that discusses numbers and calculations, discusses numerical problems, quantities, quantities, and studies the relationship between patterns, shapes and structures, and ways of thinking (Royani & Kelana, 2022). Thus, it is not uncommon for students in Indonesia to consider mathematics as a difficult subject because it deals with numbers. The important role of mathematics is that it can develop students' thinking skills, namely making students think systematically, scientifically, using logic, and critically, and can increase their creativity.

Based on the Programme for International Student Assessment (PISA) (OECD, 2019), it is stated that Indonesian students' abilities in mathematics, science, and reading are ranked low. For math alone, Indonesia is ranked 75th out of 81 countries in the world with a score of 379. Whereas in 2015, Indonesia's mathematics scores decreased to 385. With the decline in mathematics skills in Indonesia, it can be seen that the mathematics skills of Indonesian students are low, and this proves that Indonesian students are still unable to understand mathematics properly. In fact, the importance of students' understanding of mathematics is one of the objectives listed in the Regulation of the Minister of National Education of the Republic of Indonesia No. 22 of 2016 (Kementerian Pendidikan dan Kebudayaan, 2016), namely understanding mathematical concepts, explaining the relationship between concepts, and applying concepts or algorithms in problem-solving flexibly, accurately, efficiently, and precisely.

Conceptual understanding is an important basic stage in the process of learning mathematics. Students should learn mathematics with understanding (Jannah, et al., 2019), especially students with special needs. Understanding of mathematics concepts (conceptual understanding in mathematics) is the ability of students to understand concepts, operations, and relations in mathematics. Conceptual understanding is not only about what is done but why it is done (Jannah, et al., 2019). Students who can understand concepts well in the learning process will find it easier to follow the learning process and develop mathematical abilities so that students will have high learning achievement. However, students who do not understand the concept tend to find it more difficult to follow the lesson. Therefore, understanding concept needs to be taught to students (Mulyani, 2016).

One of the mathematics material taught from elementary school to college level is geometry. Geometry learning aims to understand objects that are directly related to facts, concepts, principles, skills, and their application in everyday life (Rinaldi et al., 2019). The object of geometry is an abstract object of thought, so mastery of concepts is very important. According to the results of research on elementary school students, 23% of high-ability students understand the concept of a circle (Romansyah & Nurhamdiah, 2018). As many as 43% of medium-ability students have a sufficient understanding of the concept of a circle, while 34% of students have a poor understanding of the concept. According to the results of research on understanding the concept of triangles in

elementary school students based on Van Hiele's theory, high-ability students tend to understand geometry concepts well, medium-category students have sufficient conceptual understanding, and low-category students have poor conceptual understanding (Purnawati et al., 2013).

From the results of research on the understanding of geometry concepts in elementary school students above, it can be concluded that only students who have a high level of ability have a high understanding of geometry concepts as well. Students who have a high level of ability will not experience difficulties in learning or solving mathematical problems. This is because it is influenced by various factors, one of which is interest and motivation to learn. Learning motivation is an encouragement that comes from internal and external to students who are learning so that changes in behavior occur. Students will learn effectively if they are interested in the lesson (Uno, 2017). Based on the results of interviews with one of the teachers and observations of students at SDN Pademawu Timur 5 which is an inclusive school, the main problem occurred at the school was that students' mathematics skills were still relatively low. The lowest mathematics ability is possessed by students with special needs. Based on the observation as shown in Figure 1, the students were unable to recognize two-dimensional shapes well, unable to distinguish objects in a shape, and also unable to count and recognize numbers.



Figure 1. Observation of Students at SDN Pademawu Timur 5

Extensively recorded interactions among students doing mathematics, mathematical understanding is a complex phenomenon for students doing mathematics (Pirie & Kieren, 1989). Pirie Kieren had doubts in categorizing students' understanding of mathematical concepts (Pirie, 1988). Pirie's research emphasizes the process and has several levels in the process of growth of understanding. This theory of growth in mathematical understanding is "a process that is whole, dynamic, multilevel but not linear in a recursive transcendent way and knows how students think about things beyond what is seen (Pirie & Kieren, 1994). Therefore, by knowing the flow of students' understanding development, teachers will be able to know where the difficulties experienced by students are so that teachers can help students understand so that the problem of students' lack of understanding in schools can be overcome.

There are several relevant previous studies regarding the description of concept understanding based on Pirie and Kieren's theory. The first study stated that students who have a high

level of understanding of geometric concepts fulfill the eight layers of Pirie and Kieren's theory, while students who have a moderate understanding of geometric concepts only fulfill seven layers, and students those whose understanding of geometric concepts is low only reaches one layer (Hasanah, 2019). The second research describes each layer of students' understanding in understanding the concept of sequences and series (Asih et al., 2020). Meanwhile, the third research states that high-ability students are able to fulfill all the indicators of Pirie and Kieren's theoretical layers (Suindayati et al., 2019). Therefore, based on the background of research and previous research, researchers are interested in examining the profile of conceptual understanding of students with special needs in Two-dimensional Shapes based on Pirie Kieren's theory in terms of learning motivation in elementary schools. In addition, research on understanding concepts in this study will focus on squares and rectangles to be able to describe the flow of understanding the development of student with special needs on squares and rectangles in terms of their learning motivation.

METHOD

This research uses a qualitative approach with the research design being a case study. Case study is an empirical investigation that investigates phenomena in real-life contexts, when the boundary between phenomena and context is not visible and where multiple sources of evidence are used (Sugiyono, 2014; Yin, 2003). Case study used in this research in order to explore in depth the conceptual understanding of students with special needs in two-dimensional shapes based on Pirie Kieren's theory in terms of learning motivation.

This research uses students as primary data source. Primary data is the first data recorded and collected by researchers and obtained directly from the source (Moleong, 2016). The instruments in this research were learning motivation questionnaires, concept understanding tests, and interviews. The learning motivation questionnaire in this study is a closed questionnaire type because it only needs to put a checkmark on one of the answers that are considered appropriate for it. The learning motivation questionnaire instrument consists of 20 statements that are adjusted to the indicators of learning motivation (Uno, 2017). The test instrument consists of conceptual understanding questions about square and rectangular materials, and the interview guide instrument contains a collection of customized questions that will be asked to students to find out students' understanding. Before using this research instrument, a feasibility test was carried out which was validated by the 2 validator from Madura University and the fourth-grade teacher at SD Pademawu Timur 5.

In this research, 21 fourth grade students at SDN Pademawu Timur 5 will be given a diagnostic assessment with the same level of difficulty using test questions, then students will be categorized according to the criteria of mathematical understanding ability. Furthermore, from the group of students who had the criteria for low mathematical understanding ability, 1 person was selected as the research subject. Apart from the results of the diagnostic assessment, the selection of research subjects was also carried out by involving class teachers to be interviewed about students'

abilities based on the results of daily test scores, then a mathematics test was carried out again with low level questions for the selected research subjects.

After selecting 1 student as a research subject, a learning motivation questionnaire was given to categorize the student's level of learning motivation. The range of each category is determined using statistical equations adapted to the data. The number of aspects observed is 20, so the maximum score that can be obtained is 100 and the minimum score is 20. The calculation in [equation 1](#) below aims to classify the level of learning motivation possessed by students with special needs. [Table 1](#) shows the classification of learning motivation.

$$\text{Interval} = \frac{\text{Maximum score} - \text{Minimum score}}{\text{number of aspects}} = \frac{100 - 20}{20} = 4 \tag{1}$$

Table 1. Classification of Learning Motivation

Score Range	Category
20 – 40	Very low
41 – 60	Low
61 – 80	Moderate
81 – 100	Hight

After research subject has been classified based on the table above, research subject will be given a test of understanding the concept of two-dimensional shapes to determine students' understanding of the concept and an interview will be conducted to explore the flow of the thinking process carried out when taking the concept understanding test. The interview method used in this research is a semi-structured interview method. Semi-structured interviews refer to interview guidelines in the form of questions that will be asked in the hope of obtaining adequate data regarding understanding concepts based on Pirie Kieren's theory in terms of learning motivation. If during the interview there was still missing information, the researcher asked questions outside the interview guide that had been prepared, and the interview activities were recorded via audio visual. This was done to avoid loss or reduction of information to be obtained. Data collection in this research was carried out in at least two stages. This aims to obtain valid data to describe students' conceptual understanding of squares and rectangles.

Data analysis in this research includes data reduction, data display, and conclusion drawing/verification ([Miles & Huberman, 1994](#)). Data reduction in this study consists of correcting and analyzing the test results of students' understanding of the concept of Two-Dimensional Shape using the following Pirie Kieren theory indicators as shown in [Table 2](#) below.

Table 2. Indicators of Concept Understanding Based on Pirie Kieren's Theory

No	Indicators of Concept Understanding	Pirie Kieren's Theory
1	Able to restate a concept	<i>Primitive knowing</i> (Students have a simple knowledge beginning/knowledge)
2.	Able to classify objects by certain characteristics (by concepts)	<i>Image making</i> (Students were able to make an idea of the initial knowledge he had)

3	Able to classify objects by certain characteristics (by concepts)	<i>image having</i> (students were able to make an idea of the initial knowledge he had)
4	Being able to provide examples and non-examples	<i>Property noticing</i> (Students have a view and have their own understanding)
5.	Capable of presenting concepts in various forms of mathematical representation	<i>Formalizing</i> (the student is able to develop aspects he knows to form relevant qualities.)
6.	The student is able to develop the necessary conditions or requirements of enough a concept	<i>Observing</i> (students are able to form a concept of an existing trait)
7.	Students are able to use, utilize, and select specific procedures or operations	<i>Structuring</i> (students were able to link one theorem with another and were able to give a logical reason)
8.	Students are able to apply concepts to solve problems	<i>Inventing</i> (students are able to have a complete structural understanding)

The results of student work are then transformed into the form of notes to be asked back to students, and the results of interviews on each research subject are simplified and analyzed. Furthermore, the results of interviews and data presentation were concluded in the form of a description of students' understanding of the concept of two-dimensional shape based on Pirie Kieren's theory in terms of learning motivation.

RESULTS AND DISCUSSION

From the scores of 21 students who took the diagnostic assessment, there was 1 student who had the lowest score. Then an interview was conducted with the class teacher regarding the student with the lowest score and paying attention to the results of his daily test scores. It turned out that the student with the lowest score had a low ability because he was a student with special needs who was included in the slow learner category. The test was given again with low-level questions and the results showed that the student had not been able to count and recognize numbers, which was also reinforced by the results of interviews with grade 3 and grade 2 teachers who had taught the students. The student was then selected as a research subject and named subject AZL. Subject AZL was then given a learning motivation questionnaire and obtained a score of 31 which is included in the low learning motivation category. Then subject AZL was given a concept understanding test and interview to find out his conceptual understanding of two-dimensional shape based on Pirie Kieren.

The following is an understanding of the concept of the subject AZL in Two-dimensional Shape based on Pirie Kieren's theory in terms of learning motivation:

1. Profile of Conceptual Understanding of Square in Low-Motivated Students with Special Needs Based on Pirie Kieren Theory

The results of the study showed that subject AZL had a basic understanding of points and lines in two-dimensional shapes. Conceptual understanding can be done by building new knowledge based on prior knowledge that includes dots, lines, and angles, so that conceptual understanding of students' two-dimensional shapes, especially students with special needs, can be well-formed (Jannah, et al., 2021; Jannah, et al., 2019; Jannah, et al., 2019). Subject AZL understand square and rectangular by

explaining the properties of two-dimensional shapes, namely, they both have 4 corner points and 4 sides, without knowing the difference in angles, side, and shape. Also, subject AZL confused when providing examples of buildings. [Figure 2](#) and [Figure 3](#) are the results of the tests given to subject AZL:

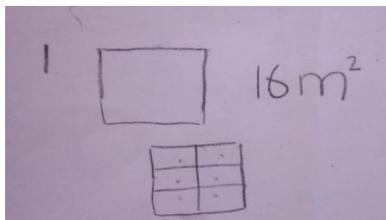


Figure 2. Results of Work on Square Material by Subject AZL



Figure 3. The Use of Plot Image Media that Fulfills the Shape to Calculate the Area

Paying attention to Pirie Kieren's layer of theoretical understanding, subject AZL can make an image/idea of the problem by drawing a square shape on the answer sheet as an example of a story problem (Image Making). This shows that there is a relationship between the type of problem and student representation ([Gokalp and Bulut, 2018](#)). Subject AZL's ability to interpret square-shaped problems shows that he understands the shape of a square and the characteristics of a square. If subject AZL is able to understand the problem, then he can explain, interpret, and apply it based on his conceptual knowledge. After that, subject AZL looked at the picture he had made and stated that the sides of the rectangle were equal in length. This shows that subject AZL is in the Property Notice layer. To find the square area, subject AZL returned to the deepest layer (Picture Making) which was assisted by first being given a stimulus about the square area because subject AZL did not know its formula.

After the subject AZL was given a stimulus, he then drew a square to find out the length of the known side of the square using the strip book media. However, subject AZL was confused because he knew that the sides were the same length, but the student could not calculate how many squares could be arranged so that the sides were the same length. This is because subject AZL cannot count and recognize numbers well. However, this shows that

when subject AZL return to the Primitive Knowledge layer, at least students have a basic understanding that a square has the same side length.

The return of the student to the deepest layers can occur when the student is faced with a problem but cannot solve it quickly (Sa'adah et al., 2020). Students have time to return to the deepest layers when facing a problem and this is called folding back (Pirie & Kieren, 1994). Back folding consists of effective back folding and ineffective back folding. Effective folding back occurs when someone can use the expanded understanding that they have gained to solve existing problems (Slaten, 2010). Ineffective back folding occurs when subject AZL are not able to use their understanding. This activity of returning to the inner level is not the same as the initial activity at that level (Gülkılıka et al., 2015). When subject AZL solved the square problem, he experienced 2 times ineffectiveness, namely in making drawings and primitive knowledge. If depicted in the form of a stacked circle, the layers of understanding of the Subject AZL in a square based on Pirie Kieren's theory shown in Figure 4 below.

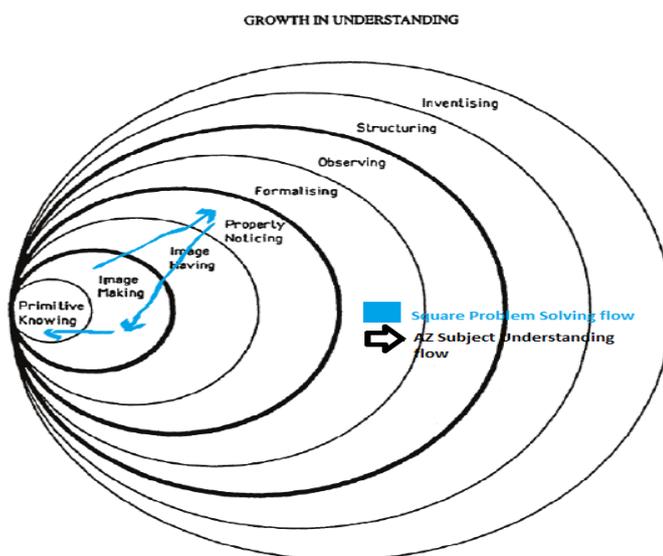


Figure 4. Layers of Understanding the Concept of Square by Subject AZL

2. Profile of Conceptual Understanding of Rectangle in Low-Motivated Students with Special Needs Based on Pirie Kieren Theory

As for the rectangle, Figure 5 shows the results of the tests that have been given to subject AZL.

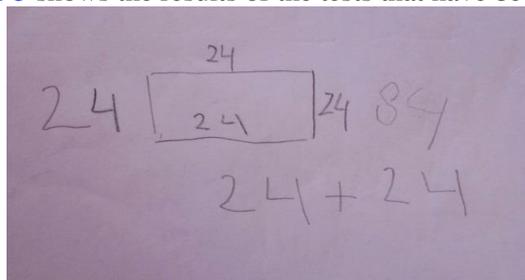


Figure 5. Results of Work on Rectangles by Subject AZL

If depicted in a nested circle, then the layers of understanding of subject AZL are as shown in Figure 6 below.

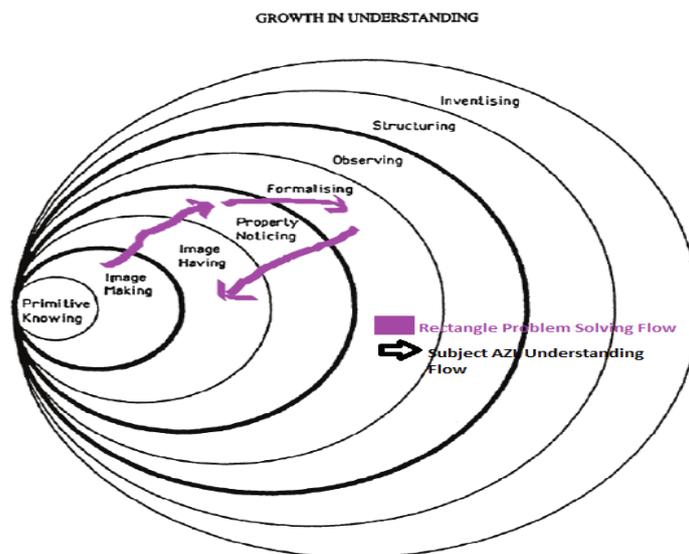


Figure 6. Layers of understanding the concept of Rectangle by Subject AZL

Based on the layers of understanding above, subject AZL drew a rectangle to illustrate the problem given (Image Making). After that, subject AZL wrote down all the side lengths 24 cm. Subject AZL wrote this because he knew the shape of a rectangle but did not know the characteristics of a rectangle, so he wrote all the side lengths were the same. In recognizing various forms of two-dimensional shapes and understanding the concepts and characteristics of students with special needs, teachers really need to do various ways to help students (Hadi, 2018). Otherwise, subject AZL will not be able to recognize various types of Two-Dimensional Shape and understand their concepts and characteristics.

Subject AZL wrote the perimeter of the rectangle by adding the length of the sides (Formalizing). From this, it can be seen that subject AZL knows how to calculate the perimeter of a rectangle and understands the meaning of the perimeter of a rectangle, but subject AZL cannot solve it because he has difficulty in calculating addition operations on numbers. In addition, subject AZL are sometimes still confused in number recognition. The same thing happened in calculating the area in question. Subject AZL could not calculate the area of the rectangle because he did not know even though he had been given a stimulus. However, subject AZL stated that the way to find the area of the rectangle was to calculate the area of the two-dimensional shape (the picture they had). The refolding movement performed by students is a need to "work on the inner layer using existing understanding" and to "collect on the inner layer" (Martin & Towers, 2016). However, AZL Subject was unable to bring his knowledge of the definition of rectangular area to calculate the rectangular area. Several studies have shown that attending to representations is essential for communication and the development of mathematical understanding (Kilpatrick et al., 2001; Taber, 2001). This form of learning is a meaningful learning that produces an organized mathematical model that allows students

with special needs to make conclusions and apply their knowledge (Jannah et al., 2021). Prior conceptual understanding of mathematics and primitive knowing is very important in understanding students' mathematical concepts (Jannah et al., 2022).

CONCLUSION

Conceptual understanding can be developed by building new knowledge based on previous knowledge which includes points, lines, and angles, so that students' conceptual understanding of two-dimensional shapes, especially students with special needs, can be well-formed. Students with special needs' understanding of squares starts from making pictures then moves on to the nature of paying attention to doing ineffective folding back to making pictures and doing ineffective folding back to primitive knowledge. Meanwhile, students with special needs understanding of rectangles start from the picture, making the properties pay attention and then folding back ineffectively on the picture.

The similarity in understanding between students with special needs in two two-dimensional shapes is that students with special needs solve the same problem, such as making a picture, and then describing the shape of the two-dimensional shape (drawing) first. Apart from that, students with special needs carry out back-folding activities because they don't know how to calculate the area. square and rectangle. Furthermore, the difference in understanding between students with special needs in two-dimensional shapes is that the refolding that students do in solving square and rectangular problems is different because when solving square problems students do 2 folding which are not effective again in making pictures and primitive knowledge, whereas in rectangles they do 1 folding which is not effective in picture. Educators are expected to be able to measure the learning motivation of students with special needs as an initial assessment before implementing learning.

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